EEE8058 Nonlinear Systems / Homework 3

Due: October 24, 2018

1. (30 pts) For the following systems, find the equilibrium points, classify them, sketch the neighboring trajectories, and try to fill in the rest of the phase portrait.

   a)
   $$\begin{aligned}\dot{x} &= x - y\\ \dot{y} &= x^2 - 4\end{aligned}$$

   b)
   $$\begin{aligned}\dot{x} &= 1 + y - e^{-x}\\ \dot{y} &= x^3 - y\end{aligned}$$

2. (20 pts) Consider the system in polar coordinates given by $\dot{r} = -r$, $\dot{\theta} = 1/\ln r$.

   a) Find $r(t)$ and $\theta(t)$ explicitly, given an initial condition (r_0, θ_0).

   b) Show that $r(t) \to 0$ and $|\theta(t)| \to \infty$ as $t \to \infty$. Therefore the origin is a stable spiral for the nonlinear system.

   c) Write the system in x, y coordinates.

   d) Show that the linearized system about the origin is $\dot{x} = -x$ and $\dot{y} = -y$. Therefore the origin is a stable star for the linearized system.

3. (15 pts) Find a conserved quantity for the system $\ddot{x} = a - e^x$ and sketch the phase portrait for $a < 0$, $a = 0$ and $a > 0$.

4. (20 pts) The model $\dot{R} = aR - bRF, \dot{F} = -cF + dRF$ is the Lotka-Volterra predator-prey model. Here $R(t)$ is the number of rabbits, $F(t)$ is the number of foxes, and $a, b, c, d > 0$ are parameters.

   a) Discuss the biological meaning of each of the terms in the model. Comment on any unrealistic assumptions.

   b) Show that the model can be recast in dimensionless form as $x' = x(1 - y)$, $y' = \mu y(x - 1)$.

   c) Find a conserved quantity in terms of the dimensionless variables.

   d) Show that the model predicts cycles in the populations of both species, for almost all initial conditions.

5. (15 pts) Show that the system $\ddot{x} + x\dot{x} + x = 0$ is reversible abnd plot the phase portrait.