EEE8058 Nonlinear Systems / Homework 4

Due: November 9, 2018

1. (20 pts) Consider the system

$$\dot{x} = y + x - x^3, \quad \dot{y} = -y$$

Find the equilibrium points, classify them, sketch the neighboring trajectories, and try to fill in the rest of the phase portrait.

2. (20 pts) Consider the system

$$\dot{x} = xy, \quad \dot{y} = x^2 - y$$

a) Show that the linearization predicts that the origin is a non-isolated equilbirium point.

b) Show that the origin is in fact an isolated equilibrium point.

c) Is the origin repelling, attracting, a saddle, or what? Sketch the vector field along the nullclines and at other points in the phase plane. Use this information to sketch the phase portrait.

d) Plot a computer-generated phase portrait to check your answer to (c).

3. (20 pts) Consider the system $\ddot{x} = x - x^2$

a) Find and classify the equilibrium points.

b) Sketch the phase portrait.

c) Find an equation for the homoclinic orbit that separates closed and nonclosed trajectories.

4. (20 pts) Consider the system $\dot{x} = xy, \quad \dot{y} = -x^2$.

a) Show that $E = x^2 + y^2$ conserved.

b) Show that the origin is a equilibrium point, but not an isolated equilibrium point.

c) Since E has a local minimum at the origin, one might have thought that the origin has to be a center. Show that in fact the origin is not surrounded by closed orbits, and sketch the actual phase portrait.

5. (20 pts) In the children's hand game of rock-paper-scissors, rock beats scissors (by smashing it); scissors beats paper (by cutting it); and paper beats rock (by covering it). In a biological setting, analogs of this non-transitive competition occur among certain types of bacteria and lizards.

   Consider the following idealized model for three competing species locked in a life-and-death game of rock-paper-scissors:

$$\dot{P} = P(R - S)$$
$$\dot{R} = R(S - P)$$
$$\dot{S} = S(P - R)$$

   where $P, R, S > 0$ are the sizes of the paper rock and scissors populations.

   a) Write a few sentences explaining the various terms in these equations. Be sure to comment on why a given term has a plus or minus sign in front of it. You don't have to write muchjust enough to demonstrate that you understand how the form of the equations reflects the rock-paper-scissors story. Also, state some of the biological assumptions being made here implicitly.

   b) Show that $P + R + S$ is a conserved quantity.

   c) Show that PRS is also conserved.

   d) How does the system behave as $t \to \infty$? Prove that your answer is correct.