EEE8058 Nonlinear Systems / Midterm Exam

Due date: 11.10.2018

1. (20 pts) For the following system with nonlinear dynamics

$$\begin{aligned}\dot{x} &= x(2 - x - y)\\ \dot{y} &= x - y\end{aligned}$$

   a) (5 pts) Find the equilibrium points.

   b) (5 pts) Sketch the nullclines (the points where the flow is completely horizontal/vertical).

   c) (5 pts) Investigate the stability of the equilibrium points .

   d) (5 pts) Sketch a plausible phase portrait (by hand and by a computer program).

2. (32 pts) The relativistic equation for the orbit of a planet around the sun is

$$\frac{d^2u}{d\theta^2} + u = \alpha + \epsilon u^2$$

   where $u = 1/r$ and r, θ are the polar coordinates of the planet in its plane of motion. The parameter α is positive and can be found explicitly from classical Newtonian mechanics; the term ϵu^2 is Einstein's correction. Here, ϵ is a very small positive parameter.

   a) (8 pts) Rewrite the equation as a system in the (u, v) phase plane where $v = du/d\theta$.

   b) (8 pts) Find all the equilibrium points of the system.

   c) (8 pts) Show that one of the equilibria is a center in the (u, v) phase plane, according to the linearization. Is it a nonlinear center?

   d) (8 pts) Show that the equilibrium point found in c) corresponds to a circular planetary orbit.

3. (32 pts) Consider the system

$$\begin{aligned}\dot{x} &= \sin y\\ \dot{y} &= \sin x\end{aligned}$$

   a) (8 pts) Show that the system is reversible.

   b) (8 pts) Find and classify all the equilibrium points.

c) (8 pts) Show that the lines $y = \pm x$ are invariant (any trajectory that starts on them stays on them forever).

d) (8 pts) Sketch the phase portrait.

4. (16 pts) Prove that the nonlinear system given by the dynamics

$$\dot{x} = -x + xy$$
$$\dot{y} = y$$

has no closed orbits.